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Logit, CES, and Rational Inattention*

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Abstract

We study fundamental links between two popular approaches to consumer choice: the multinomial logit model of individual discrete choice and the CES utility function, which describes a multiple choice of a representative consumer. We base our analysis on the rational inattention (RI) model and show that the demand system of RI agents, each of which chooses a single option, coincides with the demand system of a fictitious representative agent with CES utility function. Thus, the multiple choice of the representative agent may be explained by the heterogeneity in signals received by the RI agents. We obtain a new interpretation for the elasticity of substitution and the weighting coefficients of the CES utility function. Specifically, we provide a correspondence between parameters of the CES utility function, prior knowledge and marginal cost of information.

Keywords: discrete choice, rational inattention, CES utility function, multinomial logit, representative consumer, demand system
JEL classification codes: D40, D83, L11

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1 Introduction

People choose varieties of products for various reasons, and, perhaps, the two main reasons are variation in preferences and in information. Correspondingly, there are models of individual choice based either on heterogeneous idiosyncratic preferences or on variation in information received by agents. Both types of models have become workhorses in microeconomics, decision making and related topics. However, for the analysis of behavior of a set of consumers, rather than a single consumers, one uses an "as if" model of a fictitious representative consumer with aggregate utility function, often having the shape of a constant elasticity of substitution (CES). The existing microfoundation of the CES utility function is based exclusively on preference heterogeneity, and thus any change in parameters of the CES utility function is interpreted as a change in the idiosyncratic preferences of underlying agents, while possible informational reasons are ignored.

In the present paper we broaden the approach to the microfoundation of the CES utility function and show that this functional form might be obtained by aggregation of choices of rationally inattentive (RI) consumers who make a discrete choice with costly information acquisition. Our approach explains the origins of both the weighting coefficients (which have previously been interpreted as a consumer's preferences for separate goods) and of the elasticity of substitution of the CES utility function.

The new microfoundation is important since it allows the expansion of understanding of the concept of a representative agent and opens a way to reinterpret many models of macroeconomics, international trade and economic geography which are based on the CES utility function of the representative agent.

The multinomial logit model and the CES utility function are among the most popular tools for dealing with consumer choice problems. Despite the fact that these models use quite different assumptions (discrete individual choice and multiple choice of a representative consumer, correspondingly), there is a deep and illuminating link between them.

The existing literature which relates the CES utility function to the multinomial logit model of discrete choice is based on a random utility model (Anderson et al. (1987, 1988)). Hence, the elasticity of substitution of the aggregate utility is determined by an exogenous parameter of a specific (extreme value Gumbel) cumulative distribution function of taste dispersion. Since this parameter reflects idiosyncratic differences in preferences, it is difficult to forecast its changes under economic shocks.

The present paper, in contrast, uses rational inattention (RI) as a microfoundation and reveals the link between the parameters of the RI model, the multinomial logit, and the elasticity of substitution and weighting coefficients of the CES utility function.

We model a situation in which a consumer is facing a discrete choice problem: she possesses some income and spends it to purchase only one kind of several divisible goods. We assume that, despite the goods having certain prices, the consumer is not able to observe the prices perfectly. Limitations in consumers' attention to prices are confirmed empirically (e.g. Zeithaml (1988), Rosa-Díaz (2004)).

The assumption of uncertain prices is not crucial for our analysis. Instead we could assume that the consumer does not observe purchasable quantities perfectly. The uncertainty appears either because of prices or quantities, and we stick to uncertainty of prices for the sake of definiteness.

The RI consumer observes signals about the prices, but the structure of the signals (any joint distribution of signal and state) is itself chosen by the consumer. As usually assumed in RI models, the information is costly, and the cost of information is proportional to entropy-based reduction in uncertainty between the prior and the posterior distributions.

We explore the demand structure of RI consumers with logarithmic utility and the marginal cost of information λ. We show that this demand structure is the same as the one generated by the CES utility function (which belongs to an aggregate representative consumer) for which the elasticity of substitution is $\sigma = 1/\lambda+1$. That is, the higher the cost of information, the smaller the response of market demand to changes in prices. We show that the weighting coefficients of the CES utility function are defined by the prior knowledge of the RI consumers.

Our paper is related to several strands of literature. The microfoundation of the utility function of the representative consumer is still an open question (see Kirman (1992), Sheu (2014), Tito (2016)). It is especially important to microfound the CES shape because the CES function is used in many models of macroeconomics, international trade, economic geography and industrial organization (see, e.g., Atkin et al. (2016), Mrázová and Neary (2014), Sheu (2014)). It is notable that one of the reasons for the critique of the welfare analysis based on models with CES utility (see Kirman (1992), Tito (2016)) is that the relation between the fictitious representative consumer and real consumers is not clear and the welfare of the representative consumer seems not to be informative.

The relation between the logit model of discrete choice and the CES utility function of representative consumer was first explored by Anderson et al. (1987, 1988). They use a random utility model as a foundation for logit and show that the demand system derived from a nested logit model is also generated by the CES utility function. In particular, they show that the elasticity of substitution σ of the CES utility function is determined by a positive constant (Gumbel distribution parameter) which serves as a scale parameter of the random term in the definition of stochastic utility. More general results on connection between multinomial logit and demand systems are presented, in the same vein, by Thisse and Ushchev (2016) and by Tito (2016).

Mattsson and Weibull (2002) proposed a model of discrete choice under control costs. Here, we also construct the CES utility function within this framework and compare it with the CES function based on the RI model.

The model of RI, first introduced by Sims (1998, 2003), was applied to consumer behavior by Caplin and Dean (2015), Matějka (2015), Matějka and McKay (2012) and Tutino (2013). A foundation for the multinomial logit model based on RI is proposed by Matějka and McKay (2015).

The structure of the rest of the paper is as follows. In Section 2 we describe the RI model of consumer choice and derive from it the CES utility function of a representative consumer. In Section 3 we consider the cases of homogeneous and heterogeneous options. In Section 4 we construct the CES utility function corresponding to the Mattsson-Weibull model. Section 5 concludes and Section 6 (Appendix) contains proofs.

2 The model

There are N types of goods which are perfect substitutes for the individual consumer. The consumer is endowed with budget y which she spends entirely on one type of goods. The consumer would like to purchase the cheapest type of good to have as great a quantity of it as possible, however, at the moment of choice of the good she does not observe prices perfectly[1]. For example, there are various packages which have various prices, as well as some discounts or taxes which are not so obvious at first sight. The true payoffs related to the chosen good are revealed only after the choice is made.

Following Anderson et al. (1987), we assume that the utility of consumption of

[1] Alternatively, we could assume uncertain quantities instead of prices

good i by the individual is

$$v_i = \ln q_i, \quad i = 1, ..., N,$$

where q_i is the quantity. If the individual chooses good i to purchase, then, obviously, the consumed quantity is

$$q_i^* = \frac{y}{p_i}, \tag{1}$$

where p_i is the price, and the indirect utility is

$$V(y, p_i) = \ln\left(\frac{y}{p_i}\right). \tag{2}$$

We assume that the consumer exhausts her budget entirely. For example, that can be achieved in the following way. The buyer hands over her budget to the seller, e.g., \$10, and gets in return the amount of the good that the budget is sufficient for.

2.1 Choice of the good

Following Matějka and McKay (2015), the agent is rationally inattentive and chooses from N products characterized by utility values considered by the agent as a random vector $v = (v_1, ..., v_N)$ with distribution $G(v) \in \Delta(\mathbb{R}^N)$, where $\Delta(\mathbb{R}^N)$ is the set of all probability distributions on $\mathbb{R}^N$. More precisely, the price vector $p = (p_1, ..., p_n)$ is random, which makes $v_i = V(y, p_i)$, $(i = 1, ..., N)$ random variables. The belief about v, i.e. $G(v)$, is given exogenously by the agent's prior knowledge of prices.

The agent in principle is able to obtain ad lib precise information about the realization of the random price vector $p = (p_1, ..., p_N)$ (and, correspondingly, about the realization of the random vector of utilities $v = (v_1, ..., v_N)$). However, for the agent the information about the realization is costly. She constructs her information/action strategy in advance by solving a problem of maximization of the expected utility less the expected information cost.

The information/action strategy includes the choice of information (signal) about the realization and the choice of action (selected product) conditional on the signal. The second choice is standard: the agent simply chooses the option providing the highest expected value. The first choice is the hallmark of rational inattention. Each information/action strategy may be characterized by a vector function $(P_1(v), ..., P_N(V))$, where $P_i(v)$ is a conditional probability that product i will be chosen under the realization v . The probabilities reflect the agents choice under incomplete information, when she receives a noisy signal but does not know the realization v precisely.

It is assumed that, to reduce the uncertainty, the agent has to pay a cost $\lambda\kappa$, where $\lambda > 0$ is the marginal cost of information, and $\kappa > 0$ is the amount of information processed. The latter is the expected entropy[2] reduction between the agents prior and posterior beliefs about v. Formally, the consumers problem is described in the following way.

Consumer's problem. *The consumer's problem is to find an information processing strategy maximizing expected utility less the information cost:*

$$\max_{P_i(v)}\{\sum_{i=1}^{N}\int_v v_i P_i(v)G(dv) - \lambda\kappa(P,G)\}, \tag{3}$$

where

$$\kappa(P,G) = -\sum_{i=1}^{N} P_i^0 \ln P_i^0 + \int_v \left(\sum_{i=1}^{N} P_i(v)\ln P_i(v)\right) G(dv),$$

$P_i(v)$ *is the conditional on the realized vector* v *probability of choosing good* i, *and*

[2]The entropy of a continuous random variable X with probability density function $f(x)$ with respect to a probability measure m is $H(X) = -\int f(x)\log f(x)m(dx)$.

P_i^0 *is unconditional probability that the product of type* i *will be chosen,*

$$P_i^0 = \int_v P_i(v)G(dv), \quad i = 1, ..., N.$$

Probabilities P_i^0 are obtained as a solution of the problem (3); they reflect prior knowledge $G(v)$ and do not depend on the realization of p. However, they may depend on the marginal cost of information λ.

It is shown by Matějka and McKay (2015) that the solution, $P_i(v)$, follows the modified logit formula:

$$P_i(v) = \frac{P_i^0 e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^N P_j^0 e^{\frac{v_j}{\lambda}}}, \quad i = 1, ..., N. \tag{4}$$

By plugging (2) into (4) we obtain for the probability of choosing product i as a function of price vector and prior beliefs:

$$P_i\left(v(p)\right) = \frac{P_i^0 p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^N P_j^0 p_j^{-\frac{1}{\lambda}}}, \quad i = 1, ..., N. \tag{5}$$

The conditional expected demand for good i is $D_i = P_i(v(p))q_i^*$. Equations (1) and (5) imply the following.

Lemma. *The conditional expected demand for good* i, $D_i = P_i(v(p))q_i^*$, *is*

$$D_i = \frac{P_i^0 p_i^{-\frac{1}{\lambda}-1}}{\sum_{j=1}^N P_j^0 p_j^{-\frac{1}{\lambda}}} y, \quad i = 1, ..., N. \tag{6}$$

Thus, the market share of the good i is

$$M_i = \frac{p_i D_i}{y} = P_i^0 \left(\frac{p_i}{\mathbb{P}}\right)^{-\frac{1}{\lambda}},$$

where $\mathbb{P}$ is a price index,

$$\mathbb{P} = \left(\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}} \right)^{-\lambda}.$$

2.2 The link between rational inattention and the CES utility function

In the following proposition we show that an outside observer would see the demand of the aggregate of RI agents as if there was a fictitious representative consumer maximizing the CES utility function under full information.

Proposition. *The demand structure (6) representing the rational inattention model of discrete choice with logarithmic preferences is generated by the CES utility function*

$$U = \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{\frac{1}{\rho}},$$

which is maximized by the representative consumer subject to the budget constraint

$$\sum_{j=1}^{N} p_j q_j \leq y,$$

where the elasticity of substitution is

$$\sigma = \frac{1}{1-\rho} = \frac{1}{\lambda} + 1, \tag{7}$$

and the "weighting" coefficients are

$$\beta_i = \gamma \left(P_i^0 \right)^{1-\rho} = \gamma \left(P_i^0 \right)^{\frac{\lambda}{1+\lambda}}, \quad i = 1, ..., N, \tag{8}$$

where γ is any positive coefficient.

Proof: see the Appendix.

Thus, on the aggregate market the goods are not perfect substitutes for the representative consumer, despite being perfect substitutes for each of the underlying RI consumers.

From (7) we see that the elasticity of substitution σ is higher than 1 and depends negatively on the marginal cost of information λ. If the cost of information λ increases, then the behavior of the representative (aggregate) consumer is the same as if the elasticity of substitution went down. The reason is that the individual consumer inspects prices less, and consequently she is more likely to make errors and thus react less to changes in prices.

The weighting coefficients β_i depend positively on the corresponding unconditional probabilities P_i^0.

Corollary. *The indirect utility function of the representative consumer is*

$$\mathcal{V}(y, p_1, ..., p_N) = \gamma^{\frac{1}{\rho}} \frac{y}{\mathbb{P}},$$

where $\mathbb{P}$ *is the price index,*

$$\mathbb{P} = \left(\sum_{j=1}^{N} P_j^0 p_j^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} = \left(\sum_{j=1}^{N} P_j^0 p_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = \left(\sum_{j=1}^{N} P_j^0 p_j^{-\frac{1}{\lambda}} \right)^{-\lambda}.$$

3 Implications

3.1 The case of a priori homogeneous options

The important case is when all the options enter the prior G symmetrically, i.e. the individual does not distinguish between them before she starts processing information. Such options are referred as *a priori homogeneous.*

In this case unconditional probabilities are $P_1^0 = ... = P_N^0 = 1/N$ and conditional probabilities of choice of the goods,

$$P_i(v) = \frac{e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{N} e^{\frac{v_i}{\lambda}}}, \quad i = 1, ..., N,$$

do not depend on prior belief. This is the *multinomial logit* formula. Correspondingly,

$$P_i(v(p)) = \frac{p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^{N} p_j^{-\frac{1}{\lambda}}}, \quad i = 1, ..., N$$

and expected demands are

$$D_i = \frac{p_i^{-\frac{1}{\lambda}-1}}{\sum_{j=1}^{N} p_j^{-\frac{1}{\lambda}}} y, \quad i = 1, ..., N.$$

In the case of a priori homogeneity, as in the general case, the choice of the CES function is not determined in a unique way, but up to a constant multiplier. Natural candidates for such a CES function are two "standard" functions[3]:

$$\overline{U} = \left(\sum_{i=1}^{N} q_i^{\rho} \right)^{\frac{1}{\rho}} \tag{9}$$

and

$$\tilde{U} = \left(\sum_{i=1}^{N} \frac{1}{N} q_i^{\rho} \right)^{\frac{1}{\rho}}. \tag{10}$$

Function (9) corresponds to $\gamma = N^{1-\rho}$ in the formula (8), and function (10) corresponds to $\gamma = N^{-\rho}$. These two functions can explain the same consumer choices based on the same market data; however, they possess different properties. In particular, function (10) at the limit as $\lambda \to \infty$ converges to the Cobb-Douglas function. The function (9), in its turn, goes to infinity as $\lambda \to \infty$, what is somewhat intractable.

[3]Anderson et al. (1987, 1988) connect the logit model with function (9) but do not consider function (10).

Moreover, it is easy to show that function (10) is decreasing in the marginal cost of information, while function (9) is increasing. That is, for the representative consumer with utility function (10) an increase of the cost of information is "bad news", while for the consumer with function (9) it is "good news". This is an example of how implications do change from a singular agent level (where lower cost of information clearly leads to higher welfare) to an aggregate representative agent level. This affirms that one should be careful when using aggregate models in policy analysis.

3.2 Simple example on a priori heterogeneous options

Let us assume that a RI consumer chooses one of two goods. The first good is sold at a fixed price. The second good, in turn, is sometimes sold with a discount and sometimes has a higher price. How will such pricing affect the demand of the representative agent?

More precisely, there are two goods and two states of the world. Different goods are optimal in different states, but it is costly to identify the realization of the state of the world. The first good always has price 1. The second good costs 0.5 in the first state and 1.5 in the second state. The agent possesses prior knowledge on the probability distribution of the state of the world: g_1 and $g_2 = 1 - g_1$ are probabilities of state 1 and state 2, correspondingly. As part of her information strategy the agent obtains unconditional probabilities of choosing good 1 and good 2, P_1^0, $P_2^0 = 1 - P_1^0$, correspondingly. These probabilities depend on her prior knowledge and marginal cost of information. As is shown in formula (8), these unconditional probabilities together with the parameter of information cost determine the weighting coefficients of the CES function of the representative agent. The exact formulas and the way they are obtained can be found in the Appendix.

In the figures below we can see how exactly coefficients β_1 and β_2 change with respect to the information cost parameter λ. In Figure 1 the states of the world are equiprobable: $g_1 = g_2 = 0.5$. In Figure 2 the coefficients are depicted for the case when an agent's prior knowledge is such that $g_1 = 0.4, \quad g_2 = 0.6$. The marginal cost of information changes from 0 to 1; the blue (solid) line depicts the weighting coefficient for good 1, the red (dashed) line – for good 2.

Figure 1: Coefficients β_1 (blue solid line) and β_2 (red dashed line) dependent on λ when $g_1 = g_2 = 0.5$

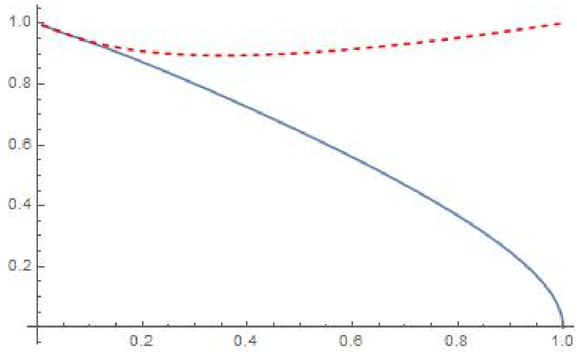
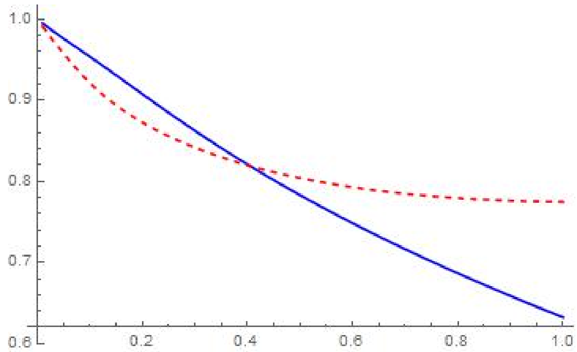


Figure 2: Coefficients β_1 (blue solid line) and β_2 (red dashed line) dependent on λ when $g_1 = 0.4, g_2 = 0.6$



We can see in Figure 1 that it is always the case that $\beta_2 > \beta_1$. It might look as if (recalling the common view on the representative consumer) for consumers the second good is intrinsically more preferable. But this is not the case – the higher weighting coefficient is explained by the information. Indeed, when we look at Figure 2, we see that for low values of λ the weighting coefficient for the first good is higher. The only thing which has changed is prior knowledge, not idiosyncratic tastes. We also see that $\beta_1 = 0$ in Figure 1 when $\lambda \geq 1$ – from the representative demand it might look like agents do not like good 1, but that is just an implication of too costly information.

This example demonstrates that the approach offered in the present paper allows us not only to provide an alternative foundation for the CES utility but also helps to understand the nature of the weighting coefficients of the CES function.

4 CES utility based on the Mattsson-Weibull model

Instead of assuming that the consumer is uncertain about prices, we could use the model of Mattsson and Weibull (2002) as a foundation of our analysis. Applying it to our story, the consumer knows the best price on the market; however, she faces control costs. In our case it could be that she knows that in a particular shop the cucumbers are the cheapest, but for that she would need to plan the visit to this shop, otherwise she might buy more expensive cucumbers in the small shop near her home.

Formally, a consumer has a "default choice rule", which is a vector of probabilities l with which she chooses goods if she makes no effort. She seeks to implement a choice m (some other choice probabilities) in order to maximize the total expected utility minus λ times the control efforts, where λ is the parameter of the marginal control cost. Formally the consumer solves:

$$\max_m \{\sum_{i=1}^N m_i q_i - \lambda \left(-\sum_{i=1}^N m_i \ln m_i + \sum_{i=1} m_i \ln l_i \right)\}.$$

The informational scarcity may be considered as one of the drivers of the control costs, and therefore the approach by Mattsson and Weibull (2002) is close in spirit to the informational approach of RI theory.

Mattsson and Weibull (2002) obtain the optimal probabilities vector m, which is, in our terms:

$$P_i(v(p)) = \frac{l_i p_i^{-\frac{1}{\lambda}}}{\sum_{j=1}^{N} l_j p_j^{-\frac{1}{\lambda}}}, \quad i = 1, ..., N.$$

Similarly to Proposition 1, we obtain the CES function of the representative consumer in the form:

$$U = \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{\frac{1}{\rho}},$$

and the weighting coefficients are

$$\beta_i = \gamma \left(l_i \right)^{1-\rho} = \gamma \left(l_i \right)^{\frac{\lambda}{1+\lambda}}, \quad i = 1, ..., N.$$

Now the elasticity of substitution is defined by the parameter of control costs, λ, but again

$$\sigma = \frac{1}{1-\rho} = \frac{1}{\lambda} + 1.$$

The higher the parameter λ is, the lower the elasticity of substitution σ.

Importantly, the weighting coefficients in this case are exogenous and determined by the default choice rule l, while in the version with RI agents the weighting coefficients are determined by unconditional probabilities which originate from maximization by the consumer.

5 Conclusion

It is often assumed that changes in the aggregate consumer's demand are due to changes in idiosyncratic preferences of individual consumers. We propose an alternative story: the demand shifts for particular goods might sometimes be better explained by a change in information about goods.

According to our model, the demand structure changes due to shifts in information costs and the structure of prior knowledge of consumers, not in the idiosyncratic preferences. In many markets there was a reduction in the costs of information (due to the appearance of websites with information on products, such as google.com/shopping, special search engines to compare the prices of airline tickets, hotels, restaurants, etc.). All this directly affects the information costs and consumer's prior beliefs. The information coming from different countries or regions and making their products salient might also change a consumer's priors. Accordingly, we can anticipate changes in the structure of the CES utility function and the aggregate consumer behavior. Thus, our model extends the understanding of why changes in demand, which are usually interpreted as a change in preferences, often occur after certain events (shocks) in the economy, such as crises, opening of new markets, and changes in the advertising policy of certain firms.

We show that the demand system generated by the CES utility function is equivalent to a model of rational inattention to discrete choice. That is, we endogenize (microfound) the CES utility function with the RI model. We show that the elasticity of substitution and "weighting" coefficients of the CES function are determined by the parameters of the RI model, namely marginal cost of information and prior beliefs. Such a link helps us to connect the intensively developing RI theory with neoclassical economic models.

Our model has several immediate implications. For example, it is known that for the market with monopolistic competition and a CES utility function of representative consumer with elasticity of substitution σ the markup equals $\sigma/(\sigma-1)$. Equation (7) implies that the markup increases with respect to the marginal cost of information λ. That is, our model contributes to the explanation of the cyclical behavior of the price-cost markup (see, e.g., Edmond and Veldkamp (2009), Nekarda and Ramey (2011, 2013). In our framework, the crisis can be characterized by an increase in information cost, which, in turn, leads to increase of markups.

Also, the results of the paper may help to find estimates for the cost of information. In the literature there are estimations of elasticities of substitution for the CES function (e.g. Bergstrand et al. (2013), Coloma (2009), Redding and Weinstein (2016)). Based on such estimations and using formula (7), which connects elasticity of substitution, σ, and marginal cost of information, λ, it is now possible to obtain the estimates for the parameter of cost of information.

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6 Appendix

6.1 Proof of the Proposition 1

Proof. Indeed, for the problem

$$\max \left(\sum_{j=1}^{N} \beta_j q_j^{\rho} \right)^{\frac{1}{\rho}}$$

s.t.

$$\sum_{j=1}^{N} p_j q_j = y, \tag{11}$$

the F.O.C. is

$$\frac{\beta_1 q_1^{\rho-1}}{p_1} = ... = \frac{\beta_N q_N^{\rho-1}}{p_N}. \tag{12}$$

From (11) and (12) it follows that

$$q_i = \frac{\beta_i^{\frac{1}{1-\rho}} p_i^{\frac{1}{\rho-1}}}{\sum_{j=1}^{N} \beta_j^{\frac{1}{1-\rho}} p_j^{\frac{\rho}{\rho-1}}} y. \tag{13}$$

By comparing (6) and (13) we see that the elasticity of substitution between goods in the CES utility function is

$$\sigma = \frac{1}{1-\rho} = \frac{1}{\lambda} + 1.$$

Correspondingly,

$$\rho = \frac{1}{\lambda + 1}.$$

Each coefficient β_i of the CES function is defined by the corresponding unconditional probability and the marginal cost of information in the following way:

$$\beta_i = \gamma \left(P_i^0\right)^{1-\rho} = \gamma \left(P_i^0\right)^{\frac{\lambda}{1+\lambda}}, \quad i = 1, ..., N,$$

where γ is a positive coefficient. $\square$

6.2 Derivation of weighting coefficients in the Example

We find the unconditional probabilities $P_i^0,\ i = 1, 2$ using the Corollary 2 from (Matějka, McKay, 2015). They should satisfy the equality:

$$\sum_{i=1}^{2} \frac{e^{\frac{v_i}{\lambda}}}{\sum_{j=1}^{2} P_j^0 e^{\frac{v_j}{\lambda}}} g_i = 1.$$

After computing the unconditional probabilities, we plug them into equation (8) and obtain the weighting coefficients of the corresponding CES function.

In our particular example under $\gamma = 1$:

$$\beta_1 = \left(P_1^0\right)^{\frac{\lambda}{1+\lambda}} = \left(\frac{-\left(\frac{4}{3}\right)^{\frac{1}{\lambda}} + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} g_1 - 2^{\frac{1}{\lambda}}(1 - g_1)}{-1 + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - \left(\frac{4}{3}\right)^{\frac{1}{\lambda}} + 2^{\frac{1}{\lambda}}} \right)^{\frac{\lambda}{1+\lambda}},$$

and

$$\beta_2 = \left(P_2^0\right)^{\frac{\lambda}{1+\lambda}} = \left(1 - \frac{-\left(\frac{4}{3}\right)^{\frac{1}{\lambda}} + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} g_1 - 2^{\frac{1}{\lambda}}(1 - g_1)}{-1 + \left(\frac{2}{3}\right)^{\frac{1}{\lambda}} - \left(\frac{4}{3}\right)^{\frac{1}{\lambda}} + 2^{\frac{1}{\lambda}}} \right)^{\frac{\lambda}{1+\lambda}}.$$

Abstrakt

Ve své práci studujeme spojitost mezi dvěma populárními metodami, které se používají při analýze spotřebitelské volby. První metodou je model individuálního diskrétního výběru zvaný multinomiální logit. Druhý přístup popisuje výběr z několika možností reprezentativního spotřebitele pomocí CES spotřebitelské funkce užitku. Svou analýzu zakládáme na teorii racionální nepozornosti (RI) a ukazujeme, že systém poptávky racionálně nepozorných spotřebitelů, kde si každý jednotlivý spotřebitel vybírá pouze jednu z dostupných možností, se shoduje s poptávkovým systémem, který modeluje reprezentativního spotřebitele pomocí CES užitkové funkce. Z výše uvedeného vyplývá, že výběr reprezentativního spotřebitele z několika dostupných možností se dá vysvětlit různorodostí signálů mezi racionálně nepozornými spotřebiteli. Z naší analýzy také vyplývá nová interpretace elasticity substituce a koeficientů vah CES užitkové funkce. Konkrétně stanovujeme korespondenci mezi parametry CES užitkové funkce, prvotními znalostmi a mezními náklady na informace.

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